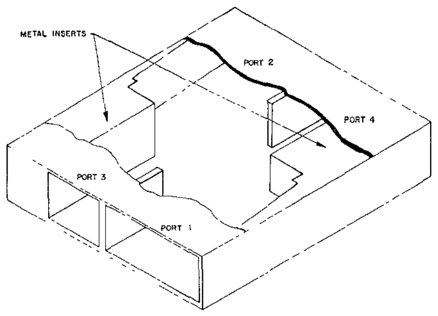

Fig. 2. Zero-dB coupler.

0.15 dB. No loads were required on ports 3 and 4, and no external matching was necessary.

A third application of the principle would consist of a compact round-to-rectangular waveguide transition.

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Pulse Power Capacity of Short-Slot Couplers

A short-slot sidewall coupler and a short-slot topwall coupler were both tested at S band for their pulse power capacity. Both types of coupler were tested in the 0-dB coupler configuration [1], which had the practical convenience of requiring only one high-power load.

In order to make meaningful tests, a special test section of S-band waveguide was constructed. This test section had the standard WR-284 waveguide width of 2.840 inches, but was only 0.447-inch high, which is one third of the standard waveguide height of 1.340 inches. A two-section waveguide transformer was placed at each end of the test section to match it to standard waveguide; the test section, including both transformers, was measured to have a maximum VSWR of 1.03 over the frequency band 2.7 to 2.9 Gc/s. All the measurements were made at a frequency of 2.856 Gc/s. The VSWR of the high-power water load at this frequency was better than 1.05. The pulse length used throughout the tests was 3 μs.

The test procedure in each case was to increase the klystron power until arcing occurred in the waveguide. The klystron power was then reduced until arcing stopped, and this power level was maintained for 10 minutes without any further arcing. Taking the test section, the sidewall coupler, and the topwall coupler in turn, the all-clear pluse power was 1.25, 2.72, and 1.67 MW, respectively. (The corresponding all-clear power averages were 0.431, 0.94, and 0.576 kW, respectively.)

Given the fact that the test section is only one third of the standard waveguide height, the pulse power capacity of the short-slot sidewall coupler is approximately 72 per cent of WR-284, and the pulse power capacity of the short-slot topwall coupler is approximately 44 per cent of WR-284 waveguide.

Acknowledgments

The authors wish to acknowledge the assistance of R. B. Larrick and P. R. Reznek in making the measurements.

The high-power tests were conducted at the Eitel-McCullough plant in San Carlos, Calif., with the cooperation of Dr. G. Caryotakis.

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Reference

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Manuscript received October 7, 1964. This work was performed under Contract No. AF 30(602)-3174 for the Rome Air Development Center.

Accurate Measurements for Dynamic Representations of Parametric Amplifier Varactor Diodes

Varactor diode parameter representations useful for describing circuit performance are usually derived from UHF or microwave measurements. Although many measurement methods have been used to date, the two main classes appear to be measurements which result in "static" and "dynamic" diode representations.

"Static" representations are defined here as those in which a UHF or microwave signal of small amplitude is used to measure characteristics of the diode which can then be expressed in terms of an equivalent circuit. UHF bridge and Q-meter [1], [2], microwave junction [3], and microwave cavity [4] measurements have been used to obtain equivalent circuits. Smith-chart data obtained from measured microwave VSWR data obtained as a function of diode bias [5], [6] or frequency [7] has been used to arrive at equivalent circuit parameters for the diode junction or cartridge. Measurements yielding circuit parameters as polynomial functions of frequency have been developed [8].

"Dynamic" representations are defined as those which characterize the diode in terms of a parameter which expresses the diode's performance under pumping conditions. It is the purpose of this note to propose a rapid and accurate measurement scheme for obtaining a dynamic quality parameter applicable to the commonly used parametric amplifier varactor.

A dynamic parameter ω_d is defined as the ratio of the total elastance variation to four times the series resistance of a pumped parametric amplifier diode [8]: $\omega_d=\Delta(1/C)/4R$. The extremes of elastance variation correspond to definite values of diode capacitance, such that if C_1 and C_2 are respectively the minimum and maximum diode capacitance, ω_d becomes

$$\omega_d = 2\pi f_d = \frac{1}{4R}\left(\frac{1}{C_1}-\frac{1}{C_2}\right) \tag{1}$$

where f_d is the "quality parameter." Let diode impedances Z_1 and Z_2 correspond to diode conditions which produced capacitances C_1 and C_2. If Z_1 and Z_2 are valid at frequency f, then

$$1 + j\,(f/2f_d) = (Z_1 + Z_2^*)/(Z_1 - Z_2) \tag{2}$$

where the asterisk denotes the complex conjugate. The measurement of Z_1 and Z_2 at the diode is not so easily accomplished, but measurements may be made when the diode terminates a transmission line. Assume that a lossless transformer is inserted between the diode and the transmission line. At the input to the transformer, the reflection coefficients Γ_1 and Γ_2 corresponding to impedances Z_1 and Z_2 may then be defined at frequency f, and f_d may be written [9] as

$$f_d = \frac{f}{2}\left[\left|\frac{1-\Gamma_1\Gamma_2^*}{\Gamma_1-\Gamma_2}\right|^2 - 1\right]^{-1/2} \tag{3}$$

Equation (3) can be rewritten in terms of the magnitudes γ_1 and γ_2, and of the phase difference Ψ between Γ_1 and Γ_2:

$$f_d = \frac{f}{2}\left[\frac{\gamma_1^2-\gamma_2^2-2\gamma_1\gamma_2\cos\Psi}{(1-\gamma_1^2)(1-\gamma_2^2)}\right]^{1/2} \tag{4}$$

Now if the lossless transformer is adjusted such that either γ_1 and γ_2 is zero, (4) can be simplified and made independent of the phase Ψ; setting $\gamma_2=0$ results in

$$f_d = \frac{f}{2}\left[\frac{1}{\gamma_1^2}-1\right]^{-1/2} \tag{5}$$

The measurement of f_d is now reduced to the measurement of a single reflection coefficient magnitude at a single frequency. For practical values of f, f_d is usually large ($f_d \gg f$) such that γ_1 is large and difficult to measure accurately using standard slotted-line techniques. However, if a reflectometer is used

Manuscript received September 21, 1964; revised October 6, 1964.

to measure the ratio of signal magnitudes returned by a standard load γ_s and the unknown γ_1, then γ_1 may be measured with reflectometer accuracy. Defining the ratio of γ_s to γ_1 as R dB,

$$R = 20 \log_{10} (\gamma_s/\gamma_1) \tag{6}$$

Standard load configurations such as a short circuit or an inductive obstacle [10] in waveguide yield a known γ_s. For example, if the standard selected is a short circuit ($\gamma_s = 1.000$), (6) becomes

$$R = 20 \log_{10} (1/\gamma_1) \tag{7}$$

Values of R vs. γ_1, as appearing in (7), have been tabulated to seven significant figures [11]. The accuracy of an f_d measurement is then a question of transformer and reflectometer errors.

Three parameters which, when non-ideal results in errors in f_d may be summarized:

1) The transformer connecting the diode to the measurement setup may be lossy, in which case (4) and (5) are incomplete.
2) The transformer may be misadjusted such that γ_2 is not equal to zero, in which case (5) is incomplete.
3) The short circuit, which permitted the simple expression (7) to be written, may be imperfect ($\gamma_s \neq 1.00$).

If a physical, lossy transformer is located ahead of the diode, then the reflection coefficients looking into the transformer will be smaller in magnitude than if the transformer were lossless. In general, losses in the transformer will depend on the physical structure of the unit, the transformation ratio N achieved, and the nature of the reflection coefficient. There is no known general method for predicting the loss in a transformer as a function of these parameters, but it is possible to measure the maximum loss of a transformer as a function of N. Defining the ratio of two-way signal transmission in the lossless case to that in the lossy case as being no greater than a certain value L, $N \leq L \geq 1$, then a reflection coefficient magnitude γ looking into the lossless transformer will be reduced to no less than γ/L looking into the lossy transformer. Assuming that the rest of the measurement system is ideal ($\gamma_2 = 0$, $\gamma_s = 1.000$), (5) now becomes

$$f_d' = \frac{f}{2}\left[\left(\frac{1}{\gamma_1'}\right)^2 - 1\right]^{-1/2} > \frac{f}{2}\left[\left(\frac{L}{\gamma_1}\right)^2 - 1\right]^{-1/2} \tag{8}$$

where γ_1' is the γ_1 measured looking into the lossy transformer. In terms of this reflection coefficient magnitude γ_1' and the maximum transformer loss L, the per-unit error $\epsilon = 1 - f_d'/f_d$ has an upper bound given by

$$\epsilon \leq 1 - \left[\frac{\frac{1}{L^2} - \gamma_1'^2}{1 - \gamma_1'^2}\right]^{1/2} \tag{9}$$

This error is always positive because $L \geq 1$, so the observed f_d' will be less than the true f_d. Equation (9) is plotted in Fig. 1 for values of L from 1.01 to 1.05. As can be expected, for small values of γ_1, loss in the transformer

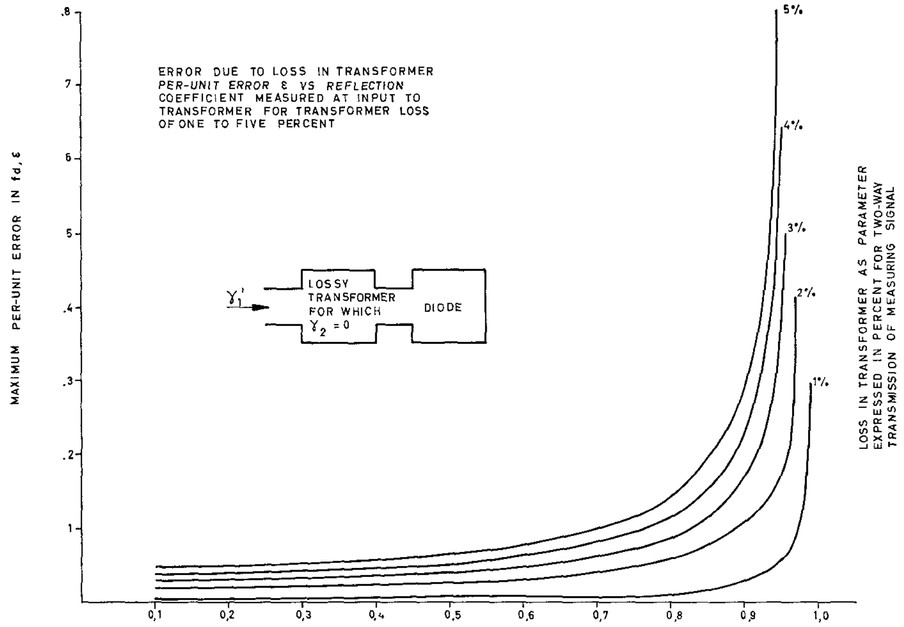

Fig. 1. Measured magnitude of γ_1 at input to transformer.

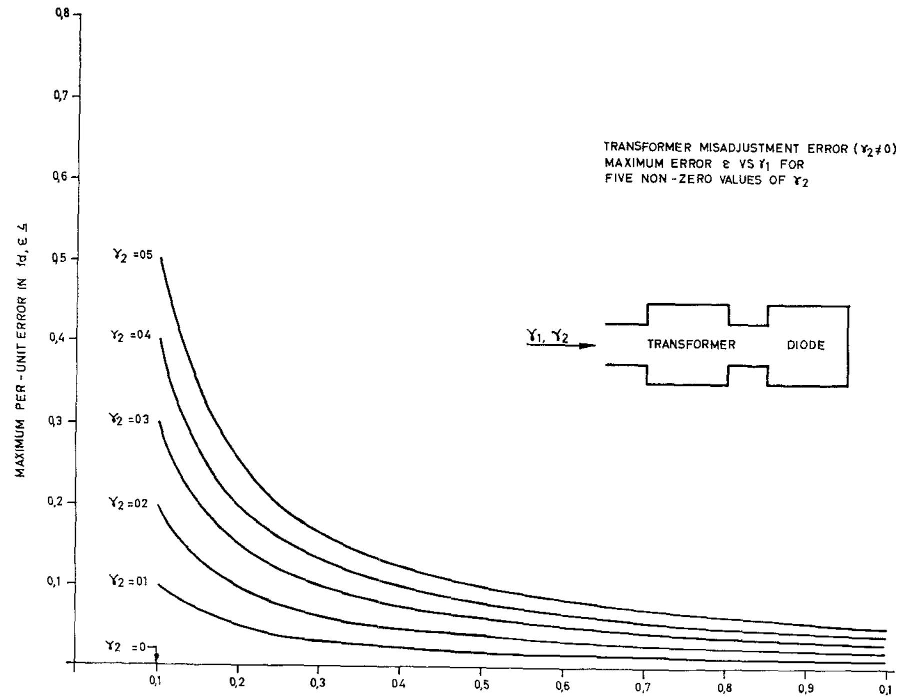

Fig. 2. Measured input reflection coefficient $\gamma_1 = |\Gamma_1|$.

produces very little error in the measurement of f_d.

If $\gamma_2 \neq 0$, but all other conditions are ideal ($L = 1.00$, $\gamma_s = 1.00$), the expression for per-unit error is bounded:

$$\epsilon \leq 1 - (1 + \gamma_2/\gamma_1)(1 - \gamma_2^2)^{-1/2} \tag{10}$$

This error is positive because $\gamma_2 \leq 1$. Equation (10) is plotted in Fig. 2 for values of γ_2 from 0.01 to 0.05.

If the short used in measuring γ_1 is imperfect but all other conditions are ideal ($L = 1.000$, $\gamma_2 = 0$), the expression for per-unit error is

$$\epsilon = 1 - [(1 - \gamma_1^2)(\gamma_s^2 - \gamma_1^2)]^{1/2} \tag{11}$$

This error is always negative, because $\gamma_s \leq 1$, thus the observed f_d' will be greater than the true f_d. Equation (11) is plotted in Fig. 3 for values of γ_s from 0.9900 to 0.9990. Equation (5)—along with Figs. 1, 2, and 3—permit f_d to be calculated and its error to be evaluated. In any event, it should be possible to hold the transformer losses below 1 per cent, adjust γ_2 to better than 0.01, and construct a short with a γ_s greater than 0.999.

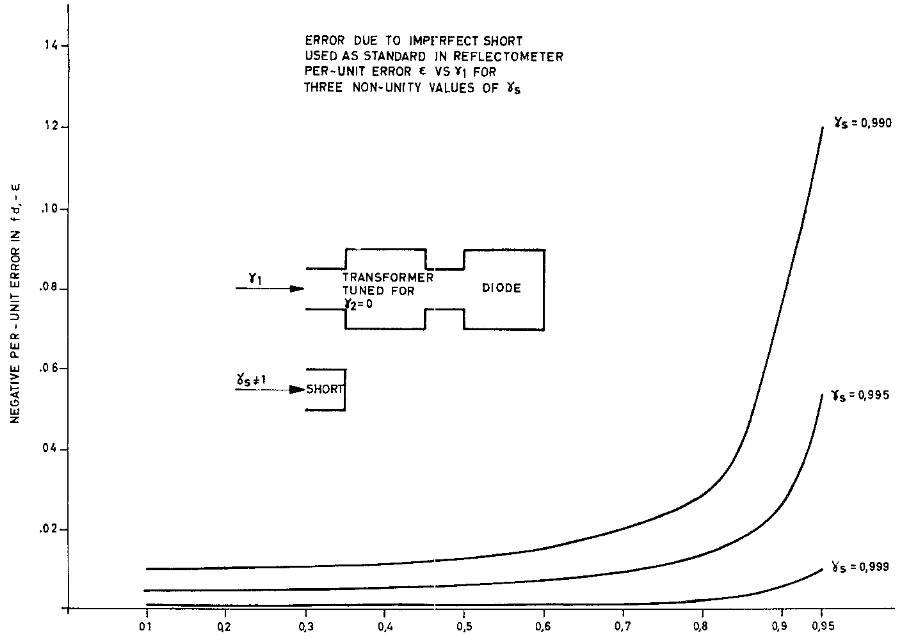

Fig. 3. Measured input reflection coefficient $\gamma_1 = |\Gamma_1|$.

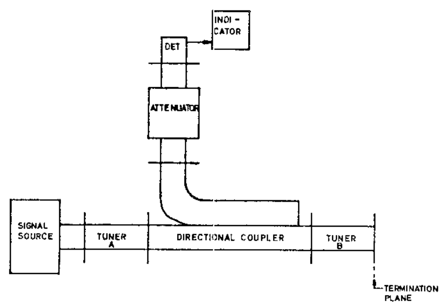

Fig. 4. Basic components of three-port reflectometer.

If this is accomplished, the error in f_d should be 5 per cent or less for values of γ_1 of 0.85 or less.

The simplicity of (7) and the subsequent error analysis are based on the assumption that the ratio γ_s/γ_1 can easily be measured as R dB. This measurement can be performed accurately with a three-port reflectometer [12], [13], shown in block form in Fig. 4. Tuners A and B are adjusted so that a match is seen looking into the termination plane and the directional coupler has nearly infinite directivity. Under these conditions, the signal incident on any load connected to the termination plane is constant. When two loads of different γ are successively connected to the termination plane, the attenuator setting must be changed by R dB to keep the detector signal constant; the ratio of the two γ is determined from (6).

The quality parameter f_d is then measured at frequency f by adjusting a well-made transformer ahead of a diode mount so that $\gamma_2 = 0$, obtaining a detector reference signal for the conditions of γ_1, and recording the differential attenuator reading R necessary to maintain this signal when the diode-mount-plus-transformer combination is replaced by a known load.

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The Characteristic Impedance and Velocity Ratio of Dielectric-Supported Strip Line

In its most practical form, strip line is made with a center conductor which consists of two thin strips of copper of the desired width on each of the two faces of a printed circuit board. Without exception known to the authors, all published theoretical results for the characteristic impedances of such lines involve the neglect of the dielectric board, i.e., the configuration analyzed is that having a center conductor of two thin unsupported strips [1].[1] The only data published to date in which the effect of the supporting card is included have been experimental [2].

In an earlier communication [3] one of the authors outlined an IBM 7090 computer program which was then being developed for the numerical analysis of TEM mode transmission lines by a finite difference approach. This program, now much enlarged and considerably accelerated, has been applied to the solution of this problem.

Theory

The strip-line configuration that was analyzed is shown in Fig. 1. The supporting card used is rexolite 2200, which has a published dielectric constant of 2.65. The object is to determine the strip width required to give a characteristic impedance of 50 ohms and the corresponding velocity ratio. The assumption is made that the system is lossless.

The relevant theory is as follows. For a selected strip width, let the capacity per unit length be computed assuming the dielectric to be absent; this is denoted by C_0. Now let the card be introduced and the capacity C per unit length recomputed. Since the inductance per unit length is clearly not changed by the presence of the dielectric, it follows that

$$Z_0 = 1/v\sqrt{CC_0} \qquad (1)$$

$$v/v_0 = \sqrt{C_0/C} \qquad (2)$$

where v is the phase velocity in the line, and v_0 is the phase velocity of light in free space.

For this work, strip widths increasing in 1/32-inch steps from 7/32 to 11/32 inch were used; the computed characteristic impedances and velocity ratios are shown in Fig. 2. From these it is concluded that a strip width of 0.279 inch must be used to obtain a 50-ohm line, and that the corresponding velocity ratio is 0.9367. From the general closeness of the velocity ratio to unity, it is seen—as would be expected—that the dielectric has a small, though significant, effect.

To test the effect of changing the dielectric material, the calculations for the 9/32-inch strip width were repeated with a dielectric constant of 2.72 (about a 3 per cent increase). The results, as summarized in

Manuscript received October 9, 1964.

[1] Since submitting this material the authors' attention has been called to the following paper. Foster, K. The characteristic impedance and phase velocity of high Q triplate line, *J. Brit. IRE*, Dec 1958, pp 715–723.